JOSEPH SCHILLINGER

KALEIDOPHONE

New Resources of Melody and Harmony

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PITCH SCALES IN RELATION TO CHORD STRUCTURES

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An Aid to

COMPOSERS • PERFORMERS • ARRANGERS • TEACHERS
SONG-WRITERS • STUDENTS • CONDUCTORS
CRITICS • AND ALL WHO WORK
WITH MUSIC

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Printed in the United States of America

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AUTHOR'S PREFACE

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THE better your sight, the more stars you see in the sky. But an unarmed eye, no matter how perfect, has its limitations. Our ancestors lived in a limited world. Ours is so great that our mind can hardly absorb what our eye can see thru a modern telescope.

Science expanded the boundaries of the universe. The visible universe of today surpasses human imagination.

You can walk and you can run; yet you would die from exhaustion and heart failure, if you were forced to run beyond your capacity. Today you can travel on land at a hundred miles per hour, and it makes you feel only a little tired.

You couldn't pull a loaded freight car even for a quarter of a mile, but an engine does it for you, carrying unimaginable loads at a super-speed.

While our ancestors had flown in their dreams, you and I can fly at four hundred miles per hour while awake, and see the curvature of this planet. If in the future we only triple this speed, we will be able to "stop the sun". Our ancestors lived literally in the dark, while our artificial lights of today make color photography possible even at night.

Although man, in many respects, is physically less developed than the animals, his brain has led him to scientific discoveries which make him the ruler of this planet. It seems natural to believe that if science provides methods which make it possible to overcome our physical limitations, it may also equip us with superhuman capacities in some other fields of human experience. Then why not apply scientific method to the arts? Creative abilities are of no different origin than any other human abilities. They all come from the same source: the brain and the nervous system. It is about time to get off the uncontrollable magic flying carpet of strained imagination and take a rocket, which will bring the stars closer to us.

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This book is a miniature reflection of a new expanding universe incorporated in my major work: "Mathematical Basis of the Arts". While the latter penetrates into all the possible forms and techniques, actual and hypothetical, this little work is only a raindrop, reflecting the immediate surroundings.

Use this book as you would use spectacles if you were near-sighted. It will relieve you from strain and despair, protect you from inhibitions and will offer you an immediate solution of many technical problems. It will open before you a new and fascinating world which is about us, yet remains unseen. It will stimulate your imagination beyond your own expectations because it will provide you with new and alluring experiences.

This book is a radical departure from the existing "musical theories". It doesn't tell you: "don't do this" or "don't do that". It overcomes completely the dualism of musical esthetic codes with their "good and evil" and "heaven and hell".

Contrary to the customary routine, which cultivates in a student fears and inhibitions, this book tells you: "don't be afraid to do as you please; this is your world, you are protected from disaster thru the very laws of this world, as you are protected from falling off this planet by the existing gravitation".

When the late George Gershwin, who, besides being a very active student of mine for four and a half years and a sincere admirer and enthusiast of my "theory", met me for the first time, he was at a dead end of creative musical experience. He felt his resources, not his abilities, were completely exhausted. He was ready to leave for Paris, where he contemplated studying with one of the leading composers. A mutual friend, Joseph Achron, who believed the study with me would save Gershwin from both a trip and a disappointment, recommended me as teacher to George.

When we met, Gershwin said: "Here is my problem: I have written about seven hundred songs. I can't write anything new any more. I am repeating myself. Can you help me?" I replied in the affirmative, and a day later, Gershwin became a sort of "Alice in Wonderland".

Later on he became acquainted with some of the materials in this book by playing them thru. "You don't have to compose music any more—it's all here," he remarked.

Every honest musician would feel the same way. While the scope of intonations in the music we have been so proud of for several centuries is confined to but a very few scales and chords, here, due to a mathematical method, are inexhaustible resources of raw materials. In a half hour of playing the tables offered in this book you get acquainted with more "master patterns" than in all the music thruout European history.

Like all the sincere seekers of truth, whether scientists or artists, who become ecstatic when they stumble upon fundamentals, George Gershwin was particularly fond of majestic simplicity which the scales disclose. Hence his extensive use of scales as thematic material, by running them up and down without shaping them into melodies, but changing their structure and the structure of the accompanying chords by means of mathematical variations. You will find much evidence of this device in "Porgy and Bess", as well as in many compositions of the last two centuries.

Whatever your ambitions and aspirations are, and whether you are contemplating a long journey or just want to take a walk around the corner —don't forget your glasses!

February 24, 1940.
New York.

BIOGRAPHICAL OUTLINE

Joseph Schillinger: Composer, lecturer, author. Born in Kharkov, Russia, Sept. 1, 1895. Head of music dept., Board of Education, Ukraine, 1918-22. Consultant to U.S.S.R. Board of Education, 1921-22. Consultant to Leningrad Board of Education, 1922-26. Professor and member State Institute of History of Arts, 1925-28.

Came to America in 1928 by invitation of American Society for Cultural Relations with Russia, to lecture on Russian contemporary music. Collaborator with Leon Theremin, 1929-32. Lecturer and instructor, 1932-36, at David Berend School of Music, Florence Cane School of Art, New School for Social Research, New York University, and in the departments of Mathematics, Music, and Fine Arts at Teachers College, Columbia University. Has exhibition of geometrical design in Mathematics Museum of Columbia University.

Evolved first scientific theory of the arts (individual and compound art forms based on the five senses, space and time), "Mathematical Basis of the Arts".

Students have included: composers, conductors, arrangers for radio and motion pictures, artists, architects, designers, and interior decorators.

Publications by the same author:

Electricity, The Liberator of Music
(Modern Music, Vol. 8, 1931)

Excerpts From A Theory of Synchronization
Experimental Cinema No. 5, 1934)

The Destiny of The Tonal Art
(Music Teachers Nat'l. Ass'n. Proceedings, American Musicological Society, 1937)

KALEIDOPHONE

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HOW TO USE THE KALEIDOPHONE

I. NOTATION

THE unit of measurement for the intervals between the different pitch-units in this system is a semitone. One octave of our tuning system consists of twelve equal semitones. The names of the pitch-units are: c, d, e, f, g, a, b. The terms "sharp", "double-sharp", "flat" and "double-flat" are added to the fundamental names. A sharp indicates that the original name is to be increased by a semitone. A double-sharp indicates that the original name is to be increased by two semitones. A flat indicates a decrease of a semitone, and a double-flat a decrease of two semitones.

The accepted musical system does not provide a notation equally as simple for all scales. Some scales appear to be more simple, some—more complicated. Scales which are the simplest in appearance are not the simplest in structure. They are merely more commonly known. Musical notation as we know it and use it, was devised to satisfy a certain type of intonations. Everything outside of such intonations often looks very complicated. The main reason for this is that many of the altered pitch-units have two, and sometimes, three names. For example, c sharp and d flat have identical intonation.

Another practical inconvenience in studying the scales is the dual terminology used for the intervals. For example, the same interval may be called a "minor third" and an "augmented second". This is why the numerical system has a definite advantage over musical terminology. Instead of two names like the "minor third" and the "augmented second", we merely use the number "3".

When measuring intervals in semitones, consider each pitch-unit a zero to which numbers are added. For example, the interval between c and f is 5, because if c itself is a zero, c sharp is 1, d is 2, e flat is 3, e natural is 4 and f is 5. This method holds true no matter how many pitch-units are

used. Let us analyze a scale: c, d, g, b flat. The interval from c to d is 2, from d to g is 5, from g to b flat is 3. The entire scale is 2+5+3.

The number of pitch-units in a scale is always one more than the number of intervals. Thus a five-unit scale has four intervals.

Musical tables correspond to numerical tables. The tables are devised in such a way that *all* the pitch-units, from which the chord is constructed, appear in *all* the *corresponding scales,* but not all of the pitch-units appear in the corresponding chords. These extra units produce the leading tones, that is, the units moving into chordal tones.

We shall call the two units, that is, the leading tone and the chordal tone into which the leading one moves, *a directional unit.* For example, in the case 17a (Triads) the chord is 4+3, that is, c, e, g and the scale—c, d flat, e, f, g. The directional units are d flat → c, d flat → e, f → e, f → g.

While the confusion due to musical terminology has been completely eliminated, the dual reading of the same pitch-unit cannot be abolished so long as we use the existing system of musical notation. The best that can be suggested is to write, wherever possible, adjacent and not identical names for the successive pitch-units. For example, in the above scale (17a) d flat is more desirable than c sharp.

II. TABLES

The main purpose the Kaleidophone tables serve, is the instantaneous locating of scales which correspond to any given chord. Such scales, and melodies which derive therefrom, are supposed to satisfy the chord. The reason for this is that all the stationary pitch-units of the scale are identical with those of the chord, while the remaining units are inserted between the stationary tones and act as leading, that is, moving tones, thus producing directional units.

The second column from the left (Table I) represents a sequence through which any chord can be immediately located. The arabic numbers of the first column on the left enumerate the chords, and the letters enumerate the corresponding scales in their consecutive order. Letters are omitted in all cases when there is only one scale to a chord.

The second column from the left represents all the chord structures in semitones. These structures cover all the possibilities within one octave

and are limited to two (diads), three (triads), four (tetrads), and five (pentads) part structures. The minimum interval in the chord structures equals 2. This permits the insertion of the leading tones *between all the adjacent chordal tones*—which does not eliminate the chords with a semitone interval between the adjacent chordal tones, as they merely appear in a different position. For example, there is no chord like c, d flat, f, a flat in these tables, but there is a 4+3+4 structure making c, e, g, b, which is the same structure in a different position.

All the chords in these tables are considered c-chords of different structures, but may be built from any other starting pitch-unit.

Depending on the numerical characteristics, different chords have a different number of corresponding scales.

Table II serves as a guide for the instantaneous locating of all chords belonging to one family (style). The latter are arranged in one horizontal row.

Each family of triads has two structures.
Each family of tetrads has either three or six chordal structures.
Each family of pentads has either four, or six, or twelve structures.
Diads are limited to one structure in a family.

These quantities depend merely on the number of possible permutations, and are more limited when there are identical intervals.

The total number of families in the class of *diads* is 10.
The total number of families in the class of *triads* is 16.
The total number of families in the class of *tetrads* is 14.
The total number of families in the class of *pentads* is 5.

As the number of units grows, the number of families diminishes. Diads are not to be considered, as they have one interval and therefore produce no variations.

The musical tables fully correspond to both numerical tables. Thus, a tetrad 36c in the numerical table, for example, expresses the same case in the musical table.

Table II offers chord progressions which, in musical notation, can be played directly on the instrument.

Table I is composed in the sequence of increasing number-values. This makes it easier to locate any chord in the table. For instance, if you open some page and see 2+3+4, while looking for 2+5+3, you have to follow it up to the point where the second place of the group becomes 5 instead of 3. This is the quickest way to find the number: locate its first numeral first, then find the second, then the next numeral and so on. In the case of 2+5+3, look for 2 first, for 5 next and for 3—last.

In Table II the numbers indicate groups, which follow the order of general permutations. The left column of this table refers to fundamental structures, where each consecutive number-value expressing an interval is greater than the preceding one.

Let us take the chord 5+2+3. In order to find the fundamental structure, we have to rearrange the numbers in their increasing value, i.e., 2+3+5. We find this structure in Table I to be case 10 in the class of tetrads. Returning to Table II, we see that the other structures of this family appear in the following order:

10 17 48 25 34 50

From Table I we learn that 5+2+3 is case 48.

The above sequence corresponds to general permutations of a group with three different terms. Numbers 2, 3 and 5 correspond to the terms a, b and c respectively.

Following is the table of six general permutations of three different terms and the corresponding cases from Table 1:

terms:	abc	acb	cab	bac	bca	cba
intervals:	235	253	523	325	352	532
cases:	10	17	48	25	34	50

Being used in such sequence as chord progressions, the above group, as well as every chord-family, produces most satisfactory results.

For the convenience of the reader I offer a full table of permutations for the cases covered in the Kaleidophone tables.

TABLE OF PERMUTATIONS

TWO-TERM GROUP

ab ba (2 permutations)

THREE-TERM GROUP

Two identical terms:

aab aba baa
or:
abb bab bba (3 permutations)

Three different terms:

abc acb cab bac bca cba (6 permutations)

FOUR-TERM GROUP

Three identical terms:

aaab aaba abaa baaa
or:
abbb babb bbab bbba (4 permutations)

Two identical pairs:

aabb abba bbaa baab abab baba (6 permutations)

Two identical terms:

aabc abca bcaa caab aacb acba cbaa baac abac baca acab caba
(12 permutations)

When the two identical terms are not in the original group, find the corresponding permutation and proceed toward the right until the end, and again from the beginning, until all the permutations are obtained.

Example: 4+2+2+1

First find the original position by assigning the two identical numbers as "a". Then 4 2 2 1 corresponds to c a a b. This is the fourth permutation out of twelve, which for this group are:

4th, 5th, 6th, 7th, 8th, 9th, 10th, 11th, 12th, 1st, 2nd and 3rd, permutations.

III. APPLICATIONS

There are numerous applications of Kaleidophone.

(1) It offers a study of all chord structures within the range of one octave. All chords are classified into 4 groups:

(a) two part structures (diads);
(b) three part structures (triads);
(c) four part structures (tetrads);
(d) five part structures (pentads).

Each group emphasizes all the possible structures. The number of chord structures usually presented in the books of harmony is very limited, and in most cases confined to European music of the XVIII Century. Thus, chords of extra-European nature which may satisfy some of the Oriental scales as well as scales for the new creative resources, are omitted.

The study of chords from these tables stimulates the development of harmonic imagination, thus broadening the outlook of the reader as to the quantity and type of chords. Some of the chords, particularly among the tetrads and pentads, produce a more disturbing effect than many of the commonly known structures. To eliminate this, redistribute the pitch-units of such chords to broader ranges. As a general rule, the best procedure to follow is to construct the chord upwards, starting with its fundamental and omitting one pitch-unit each time. For example, a chord expressed as 2+2+3+2, which produces c, d, e, g, a, may be spread vertically into the following form: c, e, a, d, g. In this form it sounds fully satisfactory and much more consonant than in its original form.

(2) It offers a study of all pitch-scales corresponding to the chords of the tables. As stated in the previous paragraph, the quantity of scales offered surpasses considerably the usual amount known to music students, who are offered a certain limited group known as major and minor scales.

A scale often serves as an impetus in composing melodies, obligatos and passages. It relieves the composer, the songwriter, or the arranger of music from habits which confine him to a few trivial intonations and gives him many new practical ideas.

Scales belonging to one family can be played or improvised upon in any desirable sequence and in any desirable melodic form.

(3) The second table offers various groups of chord progressions classified into families. A group of chord structures belonging to one family can be used in any desirable sequence. Such progressions have a cadence if the first structure is repeated at the end. This approach to chord progression cultivates the natural harmonic taste, which is a rare quality even among the great composers. It is only among such composers as Johann Sebastian

Bach, or Schubert or Wagner that one finds the workings of the instinct along these patterns. The harmonic taste more commonly observed usually produces various hybrid forms, that is, the sequences of chords often incorporate structures which have no natural relationship.

(4) Melodies can be evolved either from one given scale or from a sequence of scales belonging to one family. In both cases the melody is an outcome of a rearrangement of pitch-units in a scale in various sequences, which can be achieved either through improvisation or by means of a mathematical technique of permutations. This device offers an important impetus to any creative musician, suggesting new and original intonations which normally do not come into the sphere of his imagination because of his habits, the type of musical education acquired, and a number of other factors. Many composers and writers of songs have found that if they had only a good starting phrase they could go on further from that beginning. It is the original impetus that is most needed. There are more melodies in this world which have similar beginnings and endings, yet quite diversified middle sections, than melodies producing similarity in the middle or all the way through, which confirms the above statement.

(5) Any melody devised in the manner described in paragraph 4, may be accompanied by one chord. In actuality, this method of composing is a development of melodies from chords, which doesn't conform to the usual conception of having the melody first and the harmonization thereafter. At the same time some of the most important composers in the past very often (and some of them always) worked their melodies out from chord progressions. The most characteristic examples of such writing are found in the works of Wagner and César Franck. In some other cases the influence of harmony upon melody, though present, is not as obvious. A great number of themes composed by Beethoven, whether through melodic or harmonic approach, are of harmonic derivation as they are identical with the arpeggio forms of chords.

(6) The most elaborate use of these tables can be made in composing chord progressions from the second table, and in developing melodies in relation to such chords from the first table. This produces a more versatile type of music than one may expect, knowing that all the chords offered in this table are built from one fundamental "c".

Further uses of these tables are invaluable to musicians familiar with other technical routines such as harmony, counterpoint, etc. To them it

would open new vistas through the application of new types of chord structures and their corresponding scales, used not from one fundamental but from different starting points, making different types of chord progressions and connected through the system of voice leading (part writing). If any reader who is not familiar with harmony or counterpoint would like to use his initiative and try to accomplish the type of work just described, I can offer a simple suggestion with regard to voice leading: move each tone of the preceding chord to the nearest pitch positions of the following chord, regardless of their structure and relation in sequence. And although this method does not offer all the forms of voice leading, the nearest tone movement is in many cases the most acceptable one. With this device, and mixing the possibilities of both tables, one can actually compose, with the addition of rhythmic and instrumental development, music which would satisfy the ambitions even of a highly developed composer.

Arrangers of music can benefit from Kaleidophone as a source of new intonations in writing obligato, in developing thematically a motif taken from a song through any scales of the same family, and in reharmonizing melodic passages by new chord structures.

The resources of this book also help in improvisation as this is mostly done to progressions of chords, and all the chords a performer may encounter in the usual music are included in these tables.

Conductors who order elaborate arrangements of familiar music can benefit by suggesting new ideas and forms of development to their arrangers. Kaleidophone serves as an aid to teachers, students and music critics in helping them to establish clearer associations with regard to musical material, no matter in what form they acquired it. Often critics mistake a new intonation device, such as a scale or a chord, for a new idea in composition. The best way to use this book in the latter respect, is to translate any given melody into a new scale by reconstructing the scale and by rewriting (or playing directly on an instrument) through the direct translation of the degrees of the given scale, in the sequence in which it appears in a melody, into the corresponding degrees of a new scale.

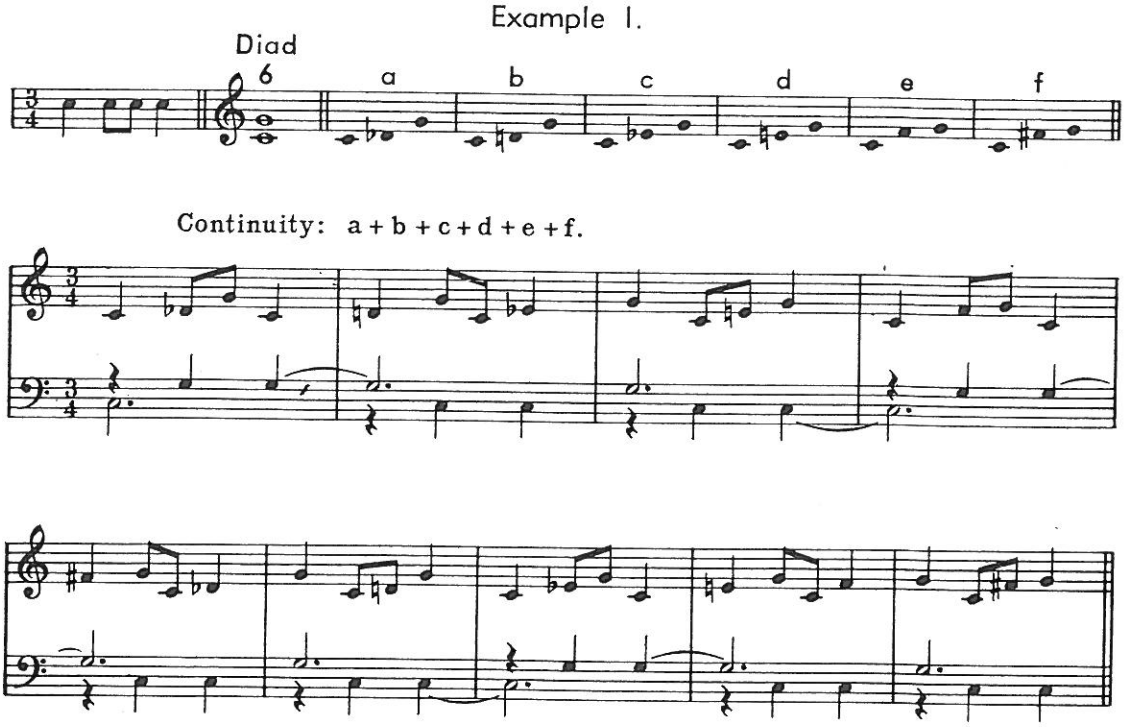
These are the fundamental uses of the tables, and many more may be developed by the reader who wants to use his own initiative.

IV. ILLUSTRATIONS FOR KALEIDOPHONE

One of the simple applications of KALEIDOPHONE is compcsition of melodic continuity from a family of scales. The simplest form of using scales is without specifying the directional units. In such a case each pitch unit participates on a par with the chordal tones. In order to specify the character of melodic continuity with harmonic accompaniment, it is necessary to pre-set a rhythmic pattern for the melody and an instrumental pattern for the harmony.

In the following example the pitch units of all scales of the family 6 of the diads appear in their original sequence, as in Table I. The harmonic accompaniment is based on varying the sequence of the lower and the upper tone of the chord structure.

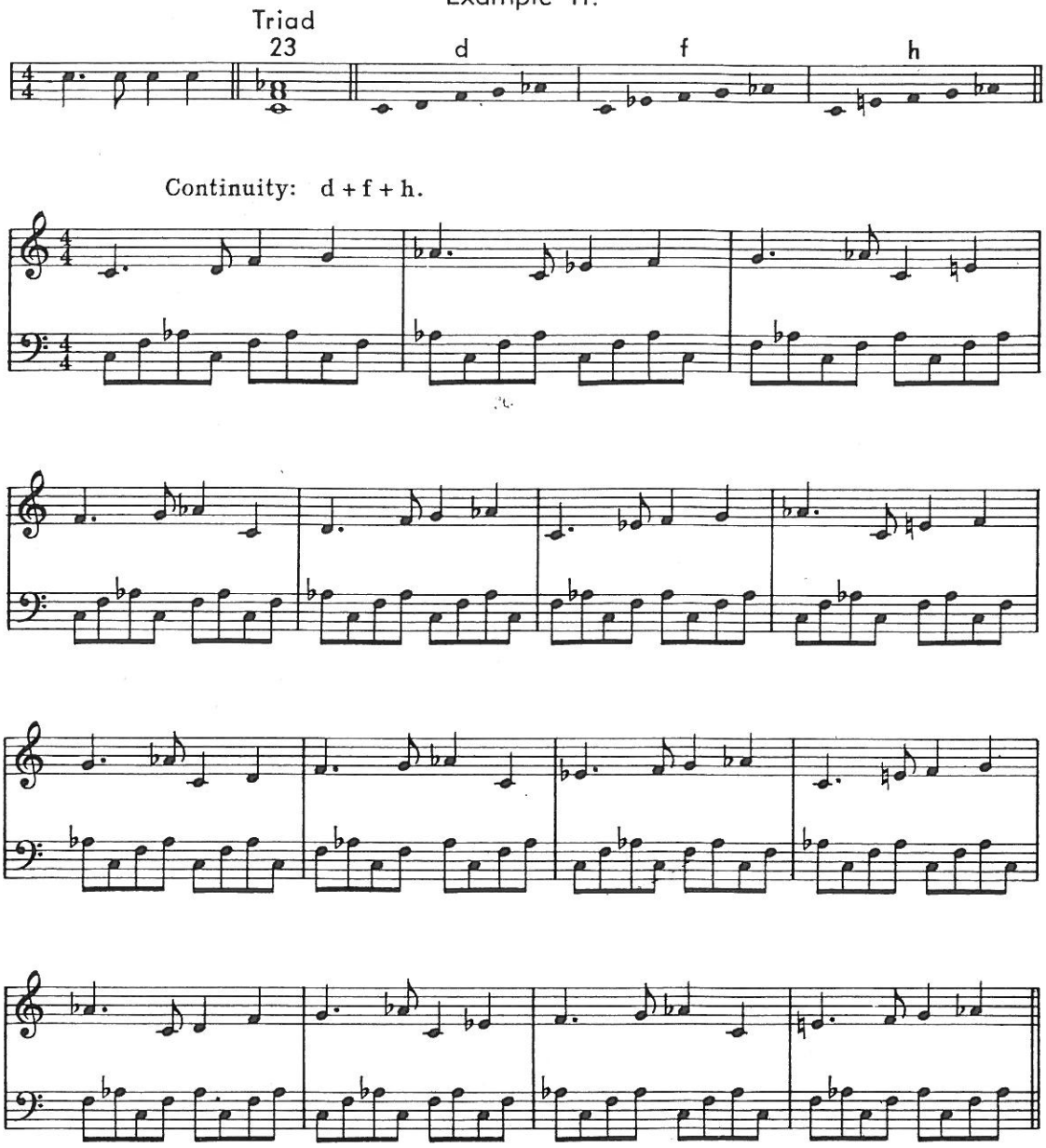
Example I.



In the following example a different rhythmic pattern is offered for the melody. The triad 23, from Table I, is used as harmony and the instru-

mental figure of it is the sequence of pitch units from lower through middle to upper. The scales chosen to form melodic continuity are d, f and h, and are used in such sequence.

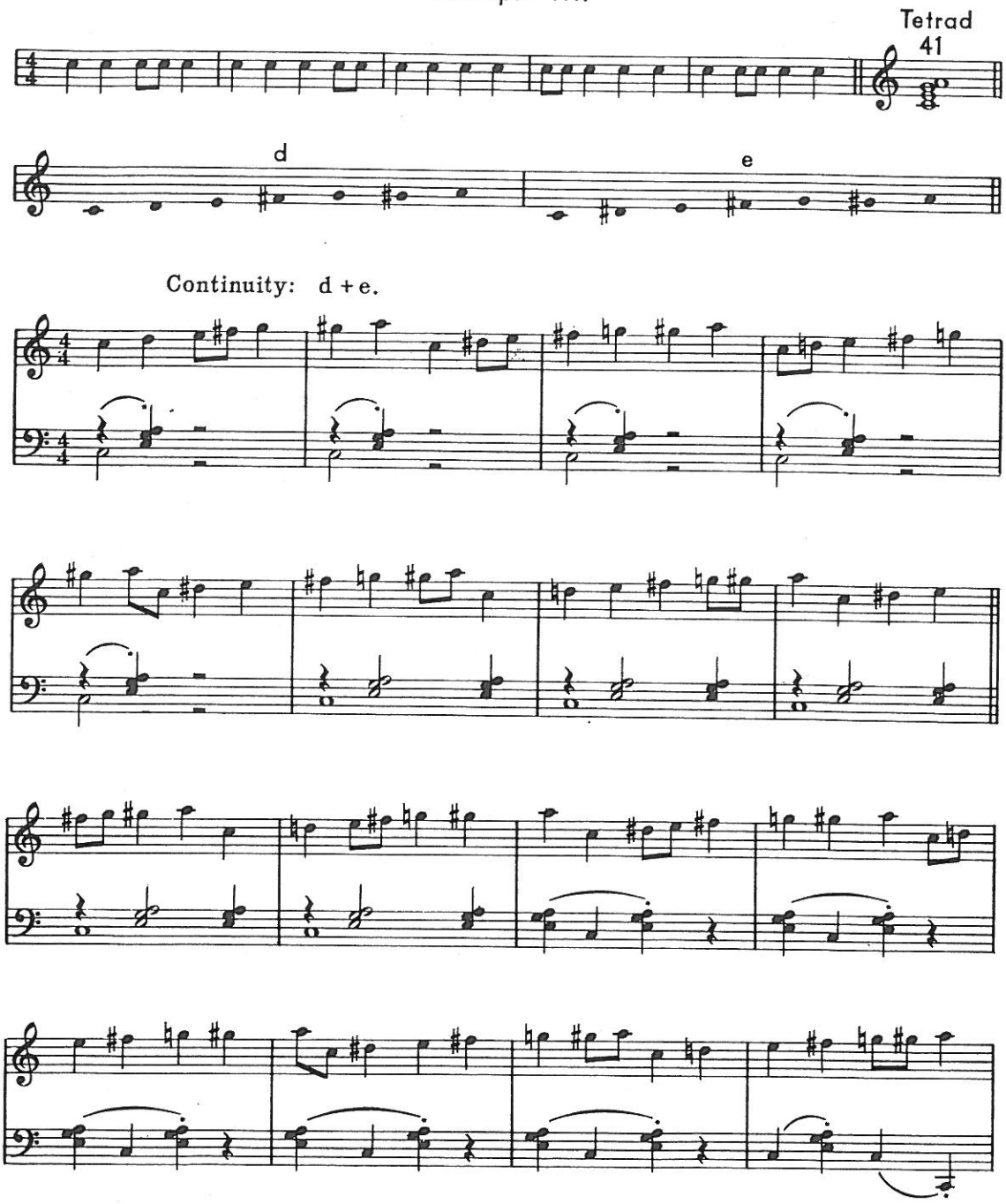
Example 11.

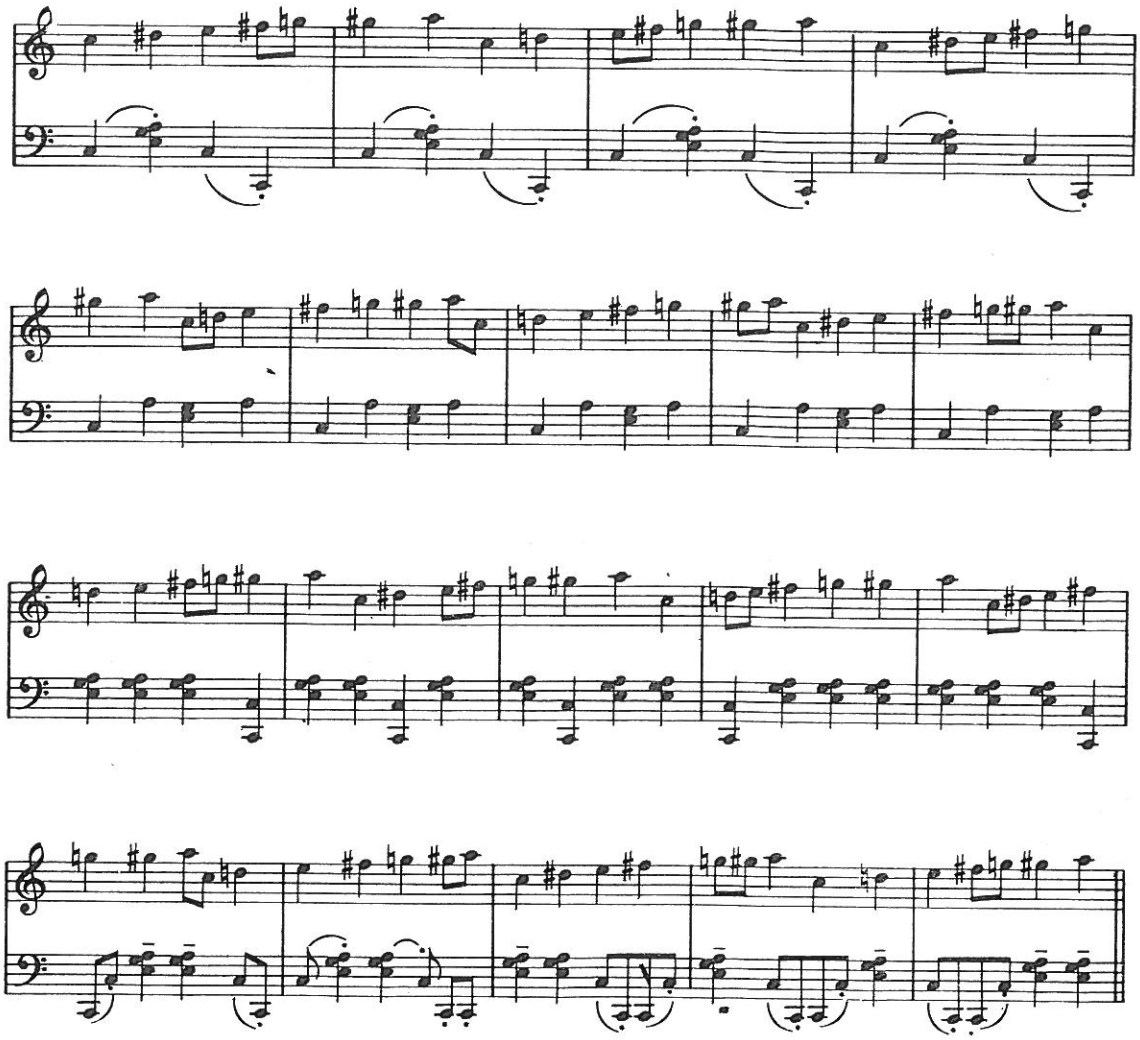


The next example illustrates a similar procedure from tetrad 41. The rhythmic pattern for the melody is extended this time to 5 measures. The

scales producing melody are limited to 2. The instrumental forms of harmonic accompaniment are varied every 5 measures. There are no repetitions until the close.

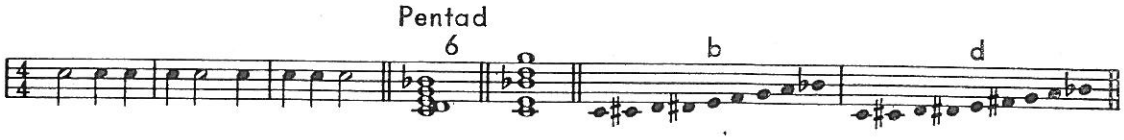
Example III.



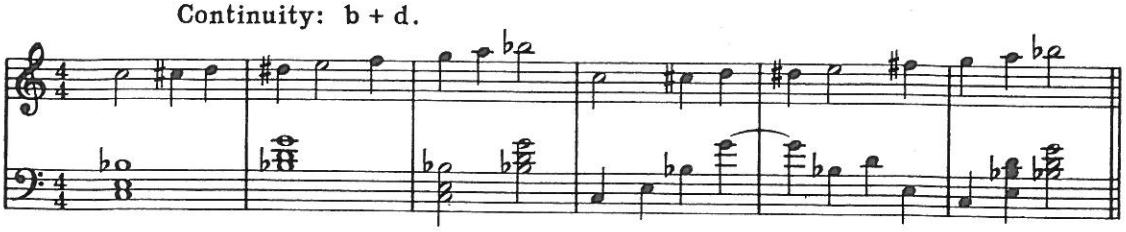


In the following example the chord structure is pentad 6. It is spread for harmonic accompaniment according to the description given in (1), page 16. The instrumental form of harmonic accompaniment is achieved by breaking up the chord in different ways. The rhythmic pattern of melody consists of 3 measures, and the melodic continuity is based on 2 scales.

Example IV.



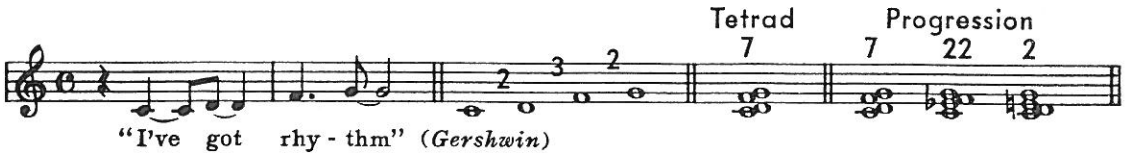
Continuity: b + d.



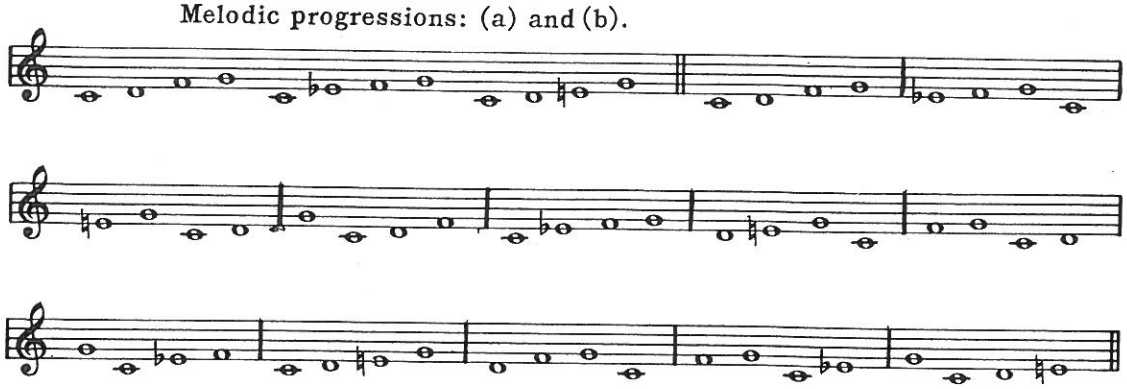
The following is an example of developing a given melodic fragment into a longer melodic continuity. Such technique may be applicable to introductions, transitions between the themes or sections of a composition, and also serve as cadenzas or improvised solos. The theme in the following example consists of 2 measures of George Gershwin's song, "I've Got Rhythm". As this melody is scalewise, it is easy to detect the scale. The latter having 4 units can be found in Table I under "Chord Structures". As the intervals of this scale are 2+3+2, it corresponds to the tetrad 7. In Table II in the "Families of Tetrads", top line of page 83, we find this structure to belong to the first family. The whole family is represented by tetrads 2, 7, 22. As the given tetrad corresponds to 7, we assume it to be the original structure and proceed our reading from that point. Thus we obtain the tetrads 7, 22, 2. Now we reconstruct this group of chords into scales appearing in the following sequence: c, d, f, g; c, e♭, f, g; c, d, e, g. As the original motif occupies 2 measures, the entire continuity becomes 6 measures long.

A more extensive melodic continuity can be obtained from the same scale material by means of permutations. Each melodic form is taken in sequence, starting every time with the following pitch unit. The original scale c, d, f, g would become d, f, g, c; f, g, d, c and g, c, d, f. If we wish to introduce further variety we can apply such permutation of pitch units to each consecutive scale of the family. Thus, the first scale appears in its original sequence of units, the second scale starting from its second pitch unit, the third scale starting from its third pitch unit, etc.

Example V.

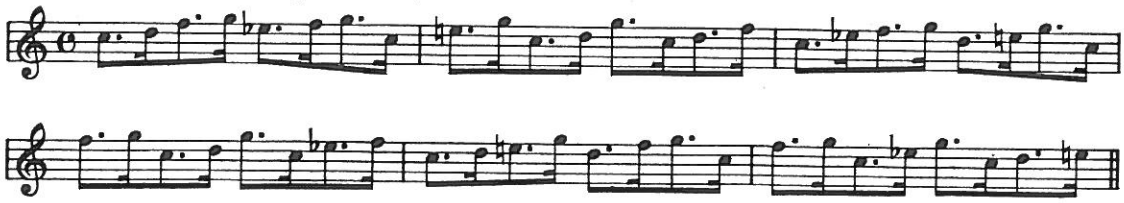


Melodic progressions: (a) and (b).



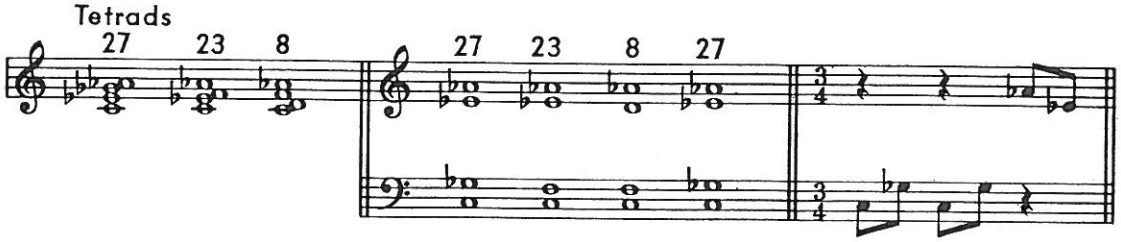
Continuity from: (a).

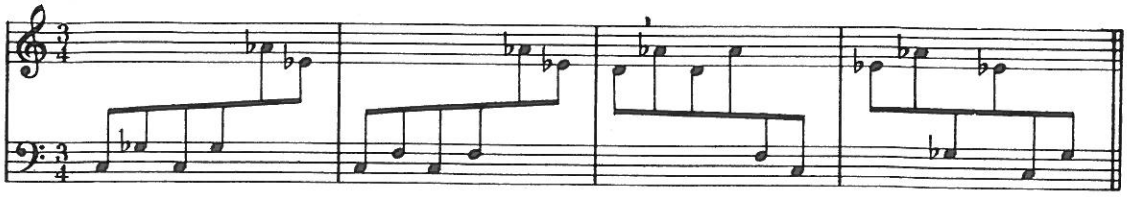
Continuity from: (b).

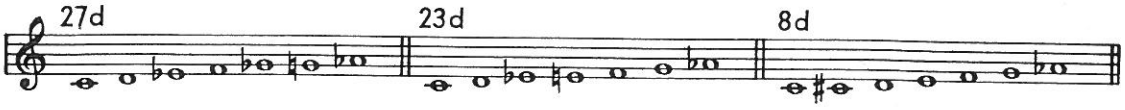


The following example illustrates the technique of developing melodic continuity to a given harmonic continuity. Our choice in this case is the family of tetrads: 27, 23 and 8. Spreading the structures and completing the sequence with the original chord, we obtain a suitable form for the instrumental development of harmonic accompaniment. Scales for melodic continuity are chosen as follows: 27d, 23d, 8d. The rhythmic pattern emphasizes 2 measures. All melodic forms are scalewise.

Example VI.

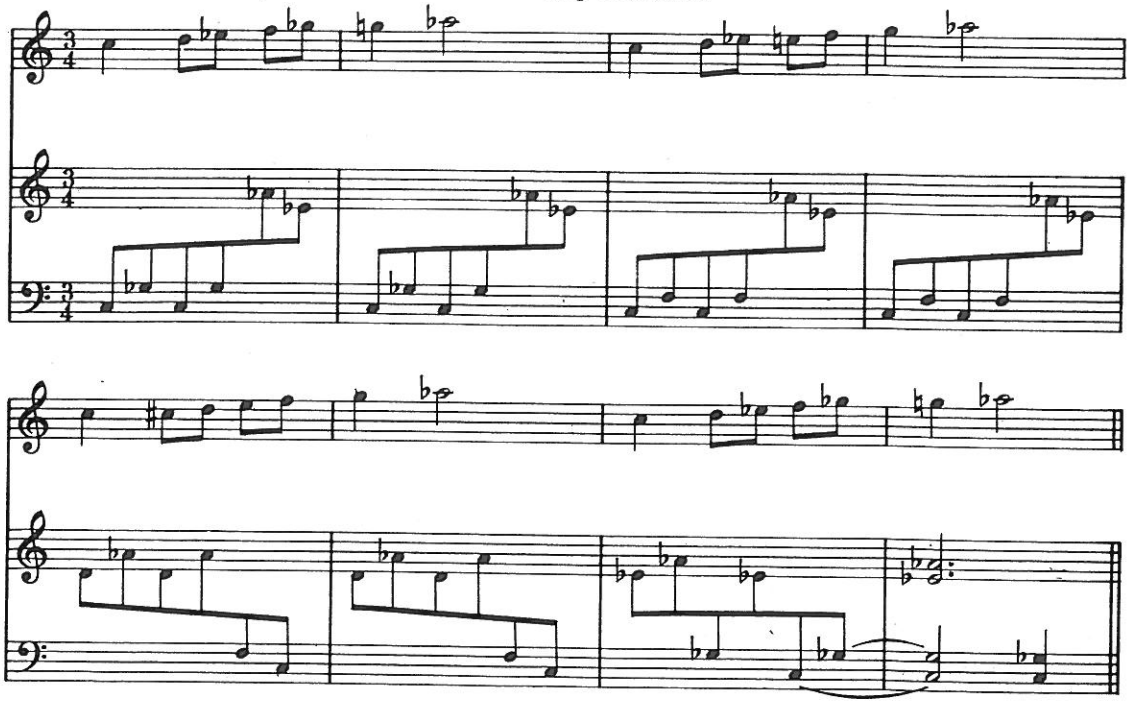






Melodic continuity: 27d + 23d + 8d.

Melody with harmonic accompaniment.

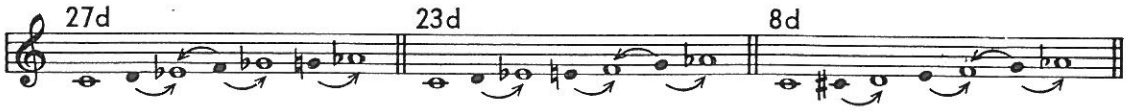


The following is a further development of the same material through specification of the directional units. As each non-chordal tone (leading tone)

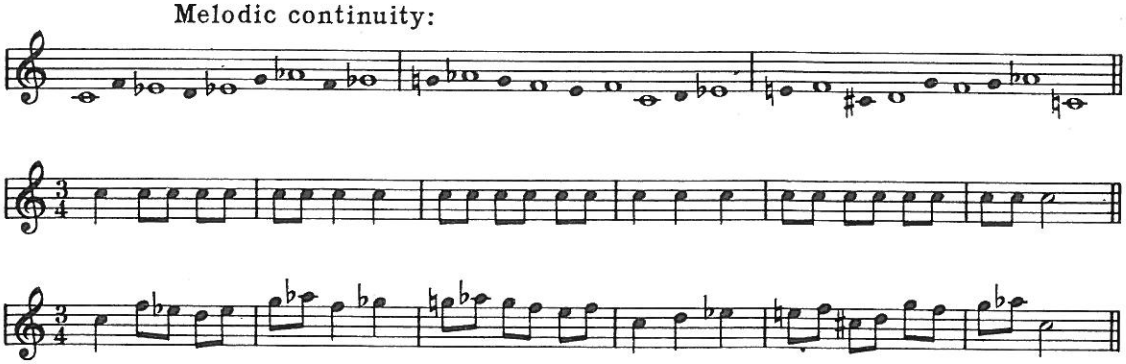
may move either to the upper or lower neighbor, our choice of the directional units in this case is an individual one. In scale 27 d, c is a neutral unit, d→e♭ is a directional unit, f→e♭ is a directional unit, f→g♭ is a directional unit and g→a♭ is a directional unit. The other 2 scales from the same selection acquire their own neutral and directional units according to the individual choice. The melodic form for each scale is also a matter of individual selection. Each of the 3 scales has a different melodic form. The entire melodic form is superimposed upon a rhythmic form which is composed with a consideration for the directional and neutral units, giving a neutral unit a relatively greater time value. The instrumental form of harmonic accompaniment is 2 measures long.

Example VII.

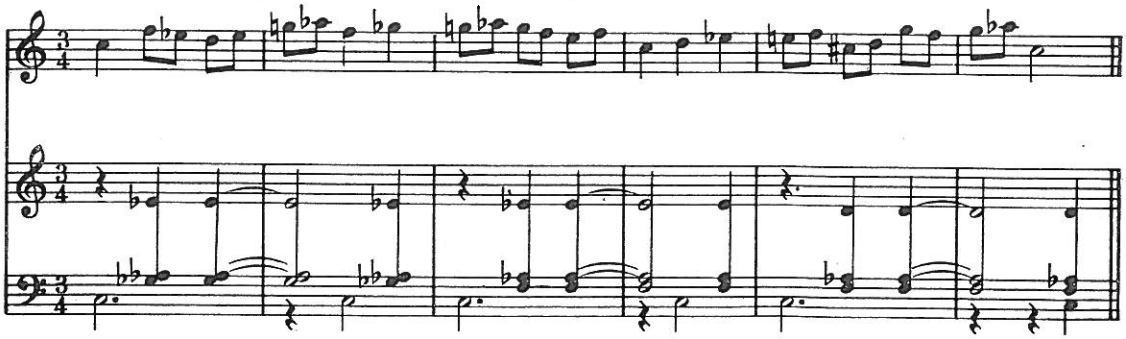
Melody with directional units from the same scale material.



Melodic continuity:



Melody with harmonic accompaniment.



KALEIDOPHONE

TABLE I

SERIAL DEVELOPMENT OF PITCH SCALES
IN RELATION TO
CHORD STRUCTURES

SERIAL DEVELOPMENT OF PITCH SCALES IN RELATION TO CHORD STRUCTURES

Arabic numerals express intervals in semitones

Chords Scales

I.

	Diads	Three Units		Diads	Three Units
1	2	1 + 1	d		4 + 2
2a	3	1 + 2	e		5 + 1
b		2 + 1	6a	7	1 + 6
3a	4	1 + 3	b		2 + 5
b		2 + 2	c		3 + 4
c		3 + 1	d		4 + 3
4a	5	1 + 4	e		5 + 2
b		2 + 3	f		6 + 1
c		3 + 2	7a	8	1 + 7
d		4 + 1	b		2 + 6
5a	6	1 + 5	c		3 + 5
b		2 + 4	d		4 + 4
c		3 + 3	e		5 + 3

	Diads	Three Units
f		6 + 2
g		7 + 1
8a	9	1 + 8
b		2 + 7
c		3 + 6
d		4 + 5
e		5 + 4
f		6 + 3
g		7 + 2
h		8 + 1
9a	10	1 + 9
b		2 + 8
c		3 + 7
d		4 + 6

	Diads	Three Units
e		5 + 5
f		6 + 4
g		7 + 3
h		8 + 2
i		9 + 1
10a	11	1 + 10
b		2 + 9
c		3 + 8
d		4 + 7
e		5 + 6
f		6 + 5
g		7 + 4
h		8 + 3
i		9 + 2
j		10 + 1

II.

	Triads	Five Units		Triads	Five Units
1	2 + 2	1+1+1+1	6a	2+7	1+1+1+6
2a	2 + 3	1+1+1+2	b		1+1+2+5
b		1+1+2+1	c		1+1+3+4
3a	2 + 4	1+1+1+3	d		1+1+4+3
b		1+1+2+2	e		1+1+5+2
c		1+1+3+1	f		1+1+6+1
4a	2 + 5	1+1+1+4	7a	2+8	1+1+1+7
b		1+1+2+3	b		1+1+2+6
c		1+1+3+2	c		1+1+3+5
d		1+1+4+1	d		1+1+4+4
5a	2 + 6	1+1+1+5	e		1+1+5+3
b		1+1+2+4	f		1+1+6+2
c		1+1+3+3	g		1+1+7+1
d		1+1+4+2	8a	2+9	1+1+1+8
e		1+1+5+1	b		1+1+2+7

	Triads	Five Units		Triads	Five Units
c		1+1+3+6	d		2+1+1+3
d		1+1+4+5	e		2+1+2+2
e		1+1+5+4	f		2+1+3+1
f		1+1+6+3	12a	3 + 5	1+2+1+4
g		1+1+7+2	b		1+2+2+3
h		1+1+8+1	c		1+2+3+2
9a	3 + 2	1+2+1+1	d		1+2+4+1
b		2+1+1+1	e		2+1+1+4
10a	3 + 3	1+2+1+2	f		2+1+2+3
b		1+2+2+1	g		2+1+3+2
c		2+1+1+2	h		2+1+4+1
d		2+1+2+1	13a	3 + 6	1+2+1+5
11a	3 + 4	1+2+1+3	b		1+2+2+4
b		1+2+2+2	c		1+2+3+3
c		1+2+3+1	d		1+2+4+2

	Triads	Five Units		Triads	Five Units
e		1+2+5+1	j		2+1+4+3
f		2+1+1+5	k		2+1+5+2
g		2+1+2+4	l		2+1+6+1
h		2+1+3+3	15a	3+8	1+2+1+7
i		2+1+4+2	b		1+2+2+6
j		2+1+5+1	c		1+2+3+5
14a	3 + 7	1+2+1+6	d		1+2+4+4
b		1+2+2+5	c		1+2+5+3
c		1+2+3+4	f		1+2+6+2
d		1+2+4+3	g		1+2+7+1
e		1+2+5+2	h		2+1+1+7
f		1+2+6+1	i		2+1+2+6
g		2+1+1+6	j		2+1+3+5
h		2+1+2+5	k		2+1+4+4
i		2+1+3+4	l		2+1+5+3

	Triads	Five Units		Triads	Five Units
m		2+1+6+2	e		2+2+2+2
n		2+1+7+1	f		2+2+3+1
16a	4 + 2	1+3+1+1	g		3+1+1+3
b		2+2+1+1	h		3+1+2+2
c		3+1+1+1	i		3+1+3+1
17a	4 + 3	1+3+1+2	19a	4+5	1+3+1+4
b		1+3+2+1	b		1+3+2+3
c		2+2+1+2	c		1+3+3+2
d		2+2+2+1	d		1+3+4+1
e		3+1+1+2	e		2+2+1+4
f		3+1+2+1	f		2+2+2+3
18a	4 + 4	1+3+1+3	g		2+2+3+2
b		1+3+2+2	h		2+2+4+1
c		1+3+3+1	i		3+1+1+4
d		2+2+1+3	j		3+1+2+3

	Triads	Five Units		Triads	Five Units
		3+1+3+2	n		3+1+4+2
l		3+1+4+1	o		3+1+5+1
20a	4 + 6	1+3+1+5	21a	4+7	1+3+1+6
b		1+3+2+4	b		1+3+2+5
c		1+3+3+3	c		1+3+3+4
d		1+3+4+2	d		1+3+4+3
e		1+3+5+1	e		1+3+5+2
f		2+2+1+5	f		1+3+6+1
g		2+2+2+4	g		2+2+1+6
h		2+2+3+3	h		2+2+2+5
i		2+2+4+2	i		2+2+3+4
j		2+2+5+1	j		2+2+4+3
k		3+1+1+5	k		2+2+5+2
l		3+1+2+4	l		2+2+6+1
m		3+1+3+3	m		3+1+1+6

	Triads	Five Units		Triads	Five Units
n		3+1+2+5	g		4+1+1+2
o		3+1+3+4	h		4+1+2+1
p		3+1+4+3	24a	5+4	1+4+1+3
q		3+1+5+2	b		1+4+2+2
r		3+1+6+1	c		1+4+3+1
22a	5 + 2	1+4+1+1	d		2+3+1+3
b		2+3+1+1	e		2+3+2+2
c		3+2+1+1	f		2+3+3+1
d		4+1+1+1	g		3+2+1+3
23a	5 + 3	1+4+1+2	h		3+2+2+2
b		1+4+2+1	i		3+2+3+1
c		2+3+1+2	j		4+1+1+3
d		2+3+2+1	k		4+1+2+2
e		3+2+1+2	l		4+1+3+1
f		3+2+2+1	25a	5+5	1+4+1+4

Triads	Five Units	Triads	Five Units
b	1+4+2+3	26a 5+6	1+4+1+5
c	1+4+3+2	b	1+4+2+4
d	1+4+4+1	c	1+4+3+3
e	2+3+1+4	d	1+4+4+2
f	2+3+2+3	e	1+4+5+1
g	2+3+3+2	f	2+3+1+5
h	2+3+4+1	g	2+3+2+4
i	3+2+1+4	h	2+3+3+3
j	3+2+2+3	i	2+3+4+2
k	3+2+3+2	j	2+3+5+1
l	3+2+4+1	k	3+2+1+5
m	4+1+1+4	l	3+2+2+4
n	4+1+2+3	m	3+2+3+3
o	4+1+3+2	n	3+2+4+2
p	4+1+4+1	o	3+2+5+1

	Triads	Five Units		Triads	Five Units
p		4+1+1+5	f		3+3+2+1
q		4+1+2+4	g		4+2+1+2
r		4+1+3+3	h		4+2+2+1
s		4+1+4+2	i		5+1+1+2
t		4+1+5+1	j		5+1+2+1
27a	6+2	1+5+1+1	29a	6+4	1+5+1+3
b		2+4+1+1	b		1+5+2+2
c		3+3+1+1	c		1+5+3+1
d		4+2+1+1	d		2+4+1+3
e		5+1+1+1	e		2+4+2+2
28a	6+3	1+5+1+2	f		2+4+3+1
b		1+5+2+1	g		3+3+1+3
c		2+4+1+2	h		3+3+2+2
d		2+4+2+1	i		3+3+3+1
e		3+3+1+2	j		4+2+1+3

	Triads	Five Units		Triads	Five Units
k		4+2+2+2	k		3+3+3+2
l		4+2+3+1	l		3+3+4+1
m		5+1+1+3	m		4+2+1+4
n		5+1+2+2	n		4+2+2+3
o		5+1+3+1	o		4+2+3+2
30a	6+5	1+5+1+4	p		4+2+4+1
b		1+5+2+3	q		5+1+1+4
c		1+5+3+2	r		5+1+2+3
d		1+5+4+1	s		5+1+3+2
e		2+4+1+4	t		5+1+4+1
f		2+4+2+3	31a	7+2	1+6+1+1
g		2+4+3+2	b		2+5+1+1
h		2+4+4+1	c		3+4+1+1
i		3+3+1+4	d		4+3+1+1
j		3+3+2+3	e		5+2+1+1

	Triads	Five Units		Triads	Five Units
f		6+1+1+1	c		1+6+3+1
32a	7+3	1+6+1+2	d		2+5+1+3
b		1+6+2+1	e		2+5+2+2
c		2+5+1+2	f		2+5+3+1
d		2+5+2+1	g		3+4+1+3
e		3+4+1+2	h		3+4+2+2
f		3+4+2+1	i		3+4+3+1
g		4+3+1+2	j		4+3+1+3
h		4+3+2+1	k		4+3+2+2
i		5+2+1+2	l		4+3+3+1
j		5+2+2+1	m		5+2+1+3
k		6+1+1+2	n		5+2+2+2
l		6+1+2+1	o		5+2+3+1
33a	7+4	1+6+1+3	p		6+1+1+3
b		1+6+2+2	q		6+1+2+2

	Triads	Five Units
r		6+1+3+1
34a	8+2	1+7+1+1
b		2+6+1+1
c		3+5+1+1
d		4+4+1+1
e		5+3+1+1
f		6+2+1+1
g		7+1+1+1
35a	8+3	1+7+1+2
b		1+7+2+1
c		2+6+1+2
d		2+6+2+1
e		3+5+1+2
f		3+5+2+1
g		4+4+1+2

	Triads	Five Units
h		4+4+2+1
i		5+3+1+2
j		5+3+2+1
k		6+2+1+2
l		6+2+2+1
m		7+1+1+2
n		7+1+2+1
36a	9+2	1+8+1+1
b		2+7+1+1
c		3+6+1+1
d		4+5+1+1
e		5+4+1+1
f		6+3+1+1
g		7+2+1+1
h		8+1+1+1

III.

	Tetrads	Seven Units
1	2 + 2 + 2	1+1+1+1+1+1
2a	2 + 2 + 3	1+1+1+1+1+2
b		1+1+1+1+2+1
3a	2 + 2 + 4	1+1+1+1+1+3
b		1+1+1+1+2+2
c		1+1+1+1+3+1
4a	2 + 2 + 5	1+1+1+1+1+4
b		1+1+1+1+2+3
c		1+1+1+1+3+2
d		1+1+1+1+4+1
5a	2 + 2 + 6	1+1+1+1+1+5
b		1+1+1+1+2+4
c		1+1+1+1+3+3

	Tetrads	Seven Units
d		1+1+1+1+4+2
e		1+1+1+1+5+1
6a	2 + 2 + 7	1+1+1+1+1+6
b		1+1+1+1+2+5
c		1+1+1+1+3+4
d		1+1+1+1+4+3
e		1+1+1+1+5+2
f		1+1+1+1+6+1
7a	2 + 3 + 2	1+1+1+2+1+1
b		1+1+2+1+1+1
8a	2 + 3 + 3	1+1+1+2+1+2
b		1+1+1+2+2+1
c		1+1+2+1+1+2
d		1+1+2+1+2+1
9a	2 + 3 + 4	1+1+1+2+1+3

Tetrads		Seven Units
b		1+1+1+2+2+2
c		1+1+1+2+3+1
d		1+1+2+1+1+3
e		1+1+2+1+2+2
f		1+1+2+1+3+1
10a	2 + 3 + 5	1+1+1+2+1+4
b		1+1+2+1+1+4
c		1+1+1+2+2+3
d		1+1+2+1+2+3
e		1+1+1+2+3+2
f		1+1+2+1+3+2
g		1+1+1+2+4+1
h		1+1+2+1+4+1
11a	2 + 3 + 6	1+1+1+2+1+5
b		1+1+1+2+2+4

	Tetrads	Seven Units
c		1+1+1+2+3+3
d		1+1+1+2+4+2
e		1+1+1+2+5+1
f		1+1+2+1+1+5
g		1+1+2+1+2+4
h		1+1+2+1+3+3
i		1+1+2+1+4+2
j		1+1+2+1+5+1
12a	2 + 4 + 2	1+1+1+3+1+1
b		1+1+2+2+1+1
c		1+1+3+1+1+1
13a	2 + 4 + 3	1+1+1+3+1+2
b		1+1+1+3+2+1
c		1+1+2+2+1+2
d		1+1+2+2+2+1

Tetrads	Seven Units
e	1+1+3+1+1+2
f	1+1+3+1+2+1
14a 2 + 4 + 4	1+1+1+3+1+3
b	1+1+1+3+2+2
c	1+1+1+3+3+1
d	1+1+2+2+1+3
e	1+1+2+2+2+2
f	1+1+2+2+3+1
g	1+1+3+1+1+3
h	1+1+3+1+2+2
i	1+1+3+1+3+1
15a 2 + 4 + 5	1+1+1+3+1+4
b	1+1+1+3+2+3
c	1+1+1+3+3+2
d	1+1+1+3+4+1

Tetrads	Seven Units
e	1+1+2+2+1+4
f	1+1+2+2+2+3
g	1+1+2+2+3+2
h	1+1+2+2+4+1
i	1+1+3+1+1+4
j	1+1+3+1+2+3
k	1+1+3+1+3+2
l	1+1+3+1+4+1
16a 2 + 5 + 2	1+1+1+4+1+1
b	1+1+2+3+1+1
c	1+1+3+2+1+1
d	1+1+4+1+1+1
17a 2 + 5 + 3	1+1+1+4+1+2
b	1+1+1+4+2+1
c	1+1+2+3+1+2

	Tetrads	Seven Units
d		1+1+2+3+2+1
e		1+1+3+2+1+2
f		1+1+3+2+2+1
g		1+1+4+1+1+2
h		1+1+4+1+2+1
18a	2 + 5 + 4	1+1+1+4+1+3
b		1+1+1+4+2+2
c		1+1+1+4+3+1
d		1+1+2+3+1+3
e		1+1+2+3+2+2
f		1+1+2+3+3+1
g		1+1+3+2+1+3
h		1+1+3+2+2+2
i		1+1+3+2+3+1
j		1+1+4+1+1+3

	Tetrads	Seven Units
k		1+1+4+1+2+2
l		1+1+4+1+3+1
19a	2 + 6 + 2	1+1+1+5+1+1
b		1+1+2+4+1+1
c		1+1+3+3+1+1
d		1+1+4+2+1+1
e		1+1+5+1+1+1
20a	2 + 6 + 3	1+1+1+5+1+2
b		1+1+1+5+2+1
c		1+1+2+4+1+2
d		1+1+2+4+2+1
e		1+1+3+3+1+2
f		1+1+3+3+2+1
g		1+1+4+2+1+2
h		1+1+4+2+2+1

	Tetrads	Seven Units
i		1+1+5+1+1+2
j		1+1+5+1+2+1
21a	2 + 7 + 2	1+1+1+6+1+1
b		1+1+2+5+1+1
c.		1+1+3+4+1+1
d		1+1+4+3+1+1
e		1+1+5+2+1+1
f		1+1+6+1+1+1
22a	3 + 2 + 2	1+2+1+1+1+1
b		2+1+1+1+1+1
23a	3 + 2 + 3	1+2+1+1+1+2
b		1+2+1+1+2+1
c		2+1+1+1+1+2
d		2+1+1+1+2+1
24a	3 + 2 + 4	1+2+1+1+1+3

	Tetrads	Seven Units
b		1+2+1+1+2+2
c		1+2+1+1+3+1
d		2+1+1+1+1+3
e		2+1+1+1+2+2
f		2+1+1+1+3+1
25a	3 + 2 + 5	1+2+1+1+1+4
b		1+2+1+1+2+3
c		1+2+1+1+3+2
d		1+2+1+1+4+1
e		2+1+1+1+1+4
f		2+1+1+1+2+3
g		2+1+1+1+3+2
h		2+1+1+1+4+1
26a	3 + 2 + 6	1+2+1+1+1+5
b		1+2+1+1+2+4

	Tetrads	Seven Units
c		1+2+1+1+3+3
d		1+2+1+1+4+2
e		1+2+1+1+5+1
f		2+1+1+1+1+5
g		2+1+1+1+2+4
h		2+1+1+1+3+3
i		2+1+1+1+4+2
j		2+1+1+1+5+1
27a	3 + 3 + 2	1+2+1+2+1+1
b		1+2+2+1+1+1
c		2+1+1+2+1+1
d		2+1+2+1+1+1
28a	3 + 3 + 3	1+2+1+2+1+2
b		1+2+1+2+2+1
c		1+2+2+1+1+2

	Tetrads	Seven Units
d		1+2+2+1+2+1
e		2+1+1+2+1+2
f		2+1+1+2+2+1
g		2+1+2+1+1+2
h		2+1+2+1+2+1
29a	3 + 3 + 4	1+2+1+2+1+3
b		1+2+1+2+2+2
c		1+2+1+2+3+1
d		1+2+2+1+1+3
e		1+2+2+1+2+2
f		1+2+2+1+3+1
g		2+1+1+2+1+3
h		2+1+1+2+2+2
i		2+1+1+2+3+1
j		2+1+2+1+1+3

	Tetrads	Seven Units
k		2+1+2+1+2+2
l		2+1+2+1+3+1
30a	3 + 3 + 5	1+2+1+2+1+4
b		1+2+1+2+2+3
c		1+2+1+2+3+2
d		1+2+1+2+4+1
e		1+2+2+1+1+4
f		1+2+2+1+2+3
g		1+2+2+1+3+2
h		1+2+2+1+4+1
i		2+1+1+2+1+4
j		2+1+1+2+2+3
k		2+1+1+2+3+2
l		2+1+1+2+4+1
m		2+1+2+1+1+4

	Tetrads	Seven Units
n		2+1+2+1+2+3
o		2+1+2+1+3+2
p		2+1+2+1+4+1
31a	3 + 4 + 2	1+2+1+3+1+1
b		1+2+2+2+1+1
c		1+2+3+1+1+1
d		2+1+1+3+1+1
e		2+1+2+2+1+1
f		2+1+3+1+1+1
32a	3 + 4 + 3	1+2+1+3+1+2
b		1+2+2+2+1+2
c		1+2+3+1+1+2
d		1+2+1+3+2+1
e		1+2+2+2+2+1
f		1+2+3+1+2+1

	Tetrads	Seven Units
g		2+1+1+3+1+2
h		2+1+2+2+1+2
i		2+1+3+1+1+2
j		2+1+1+3+2+1
k		2+1+2+2+2+1
l		2+1+3+1+2+1
33a	3 + 4 + 4	1+2+1+3+1+3
b		2+1+1+3+1+3
c		1+2+1+3+2+2
d		2+1+1+3+2+2
e		1+2+1+3+3+1
f		2+1+1+3+3+1
g		1+2+2+2+1+3
h		2+1+2+2+1+3
l		1+2+2+2+2+2

Tetrads		Seven Units
j		2+1+2+2+2+2
k		1+2+2+2+3+1
l		2+1+2+2+3+1
m		1+2+3+1+1+3
n		2+1+3+1+1+3
o		1+2+3+1+2+2
p		2+1+3+1+2+2
q		1+2+3+1+3+1
r		2+1+3+1+3+1
34a	3 + 5 + 2	1+2+1+4+1+1
b		1+2+2+3+1+1
c		1+2+3+2+1+1
d		1+2+4+1+1+1
e		2+1+1+4+1+1
f		2+1+2+3+1+1

	Tetrads	Seven Units
g		2+1+3+2+1+1
h		2+1+4+1+1+1
35a	3 + 5 + 3	1+2+1+4+1+2
b		1+2+2+3+1+2
c		1+2+3+2+1+2
d		1+2+4+1+1+2
e		1+2+1+4+2+1
f		1+2+2+3+2+1
g		1+2+3+2+2+1
h		1+2+4+1+2+1
i		2+1+1+4+1+2
j		2+1+2+3+1+2
k		2+1+3+2+1+2
l		2+1+4+1+1+2
m		2+1+1+4+2+1

	Tetrads	Seven Units
n		2+1+2+3+2+1
o		2+1+3+2+2+1
p		2+1+4+1+2+1
36a	3 + 6 + 2	1+2+1+5+1+1
b		1+2+2+4+1+1
c		1+2+3+3+1+1
d		1+2+4+2+1+1
e		1+2+5+1+1+1
f		2+1+1+5+1+1
g		2+1+2+4+1+1
h		2+1+3+3+1+1
i		2+1+4+2+1+1
j		2+1+5+1+1+1
37a	4 + 2 + 2	1+3+1+1+1+1
b		2+2+1+1+1+1

	Tetrads	Seven Units
c		3+1+1+1+1+1
38a	4 + 2 + 3	1+3+1+1+1+2
b		1+3+1+1+2+1
c		2+2+1+1+1+2
d		2+2+1+1+2+1
e		3+1+1+1+1+2
f		3+1+1+1+2+1
39a	4 + 2 + 4	1+3+1+1+1+3
b		1+3+1+1+2+2
c		1+3+1+1+3+1
d		2+2+1+1+1+3
e		2+2+1+1+2+2
f		2+2+1+1+3+1
g		3+1+1+1+1+3
h		3+1+1+1+2+2

	Tetrads	Seven Units
l		3+1+1+1+3+1
40a	4 + 2 + 5	1+3+1+1+1+4
b		1+3+1+1+2+3
c		1+3+1+1+3+2
d		1+3+1+1+4+1
e		2+2+1+1+1+4
f		2+2+1+1+2+3
g		2+2+1+1+3+2
h		2+2+1+1+4+1
i		3+1+1+1+1+4
j		3+1+1+1+2+3
k		3+1+1+1+3+2
l		3+1+1+1+4+1
41a	4 + 3 + 2	1+3+1+2+1+1
b		1+3+2+1+1+1

	Tetrads	Seven Units
c		2+2+1+2+1+1
d		2+2+2+1+1+1
e		3+1+1+2+1+1
f		3+1+2+1+1+1
42a	4 + 3 + 3	1+3+1+2+1+2
b		2+2+1+2+1+2
c		3+1+1+2+1+2
d		1+3+1+2+2+1
e		2+2+1+2+2+1
f		3+1+1+2+2+1
g		1+3+2+1+1+2
h		2+2+2+1+1+2
i		3+1+2+1+1+2
j		1+3+2+1+2+1
k		2+2+2+1+2+1

	Tetrads	Seven Units
l		3+1+2+1+2+1
43a	4 + 3 + 4	1+3+1+2+1+3
b		1+3+2+1+1+3
c		1+3+1+2+2+2
d		1+3+2+1+2+2
e		1+3+1+2+3+1
f		1+3+2+1+3+1
g		2+2+1+2+1+3
h		2+2+2+1+1+3
i		2+2+1+2+2+2
j		2+2+2+1+2+2
k		2+2+1+2+3+1
l		2+2+2+1+3+1
m		3+1+1+2+1+3
n		3+1+2+1+1+3

	Tetrads	Seven Units
o		3+1+1+2+2+2
p		3+1+2+1+2+2
q		3+1+1+2+3+1
r		3+1+2+1+3+1
44a	4 + 4 + 2	1+3+1+3+1+1
b		1+3+2+2+1+1
c		1+3+3+1+1+1
d		2+2+1+3+1+1
e		2+2+2+2+1+1
f		2+2+3+1+1+1
g		3+1+1+3+1+1
h		3+1+2+2+1+1
i		3+1+3+1+1+1
45a	4 + 4 + 3	1+3+1+3+1+2
b		1+3+1+3+2+1

Tetrads	Seven Units
c	1+3+2+2+1+2
d	1+3+2+2+2+1
e	1+3+3+1+1+2
f	1+3+3+1+2+1
g	2+2+1+3+1+2
h	2+2+1+3+2+1
i	2+2+2+2+1+2
j	2+2+2+2+2+1
k	2+2+3+1+1+2
l	2+2+3+1+2+1
m	3+1+1+3+1+2
n	3+1+1+3+2+1
o	3+1+2+2+1+2
p	3+1+2+2+2+1
q	3+1+3+1+1+2

	Tetrads	Seven Units
r		3+1+3+1+2+1
46a	4 + 5 + 2	1+3+1+4+1+1
b		1+3+2+3+1+1
c		1+3+3+2+1+1
d		1+3+4+1+1+1
e		2+2+1+4+1+1
f		2+2+2+3+1+1
g		2+2+3+2+1+1
h		2+2+4+1+1+1
i		3+1+1+4+1+1
j		3+1+2+3+1+1
k		3+1+3+2+1+1
l		3+1+4+1+1+1
47a	5 + 2 + 2	1+4+1+1+1+1
b		2+3+1+1+1+1

Tetrads	Seven Units
c	3+2+1+1+1+1
d	4+1+1+1+1+1
48a 5 + 2 + 3	1+4+1+1+1+2
b	1+4+1+1+2+1
c	2+3+1+1+1+2
d	2+3+1+1+2+1
e	3+2+1+1+1+2
f	3+2+1+1+2+1
g	4+1+1+1+1+2
h	4+1+1+1+2+1
49a 5 + 2 + 4	1+4+1+1+1+3
b	1+4+1+1+2+2
c	1+4+1+1+3+1
d	2+3+1+1+1+3
e	2+3+1+1+2+2

	Tetrads	Seven Units
f		2+3+1+1+3+1
g		3+2+1+1+1+3
h		3+2+1+1+2+2
i		3+2+1+1+3+1
j		4+1+1+1+1+3
k		4+1+1+1+2+2
l		4+1+1+1+3+1
50a	5 + 3 + 2	1+4+1+2+1+1
b		1+4+2+1+1+1
c		2+3+1+2+1+1
d		2+3+2+1+1+1
e		3+2+1+2+1+1
f		3+2+2+1+1+1
g		4+1+1+2+1+1
h		4+1+2+1+1+1

	Tetrads	Seven Units
51a	5 + 3 + 3	1+4+1+2+1+2
b		2+3+1+2+1+2
c		3+2+1+2+1+2
d		4+1+1+2+1+2
e		1+4+1+2+2+1
f		2+3+1+2+2+1
g		3+2+1+2+2+1
h		4+1+1+2+2+1
i		1+4+2+1+1+2
j		2+3+2+1+1+2
k		3+2+2+1+1+2
l		4+1+2+1+1+2
m		1+4+2+1+2+1
n		2+3+2+1+2+1
o		3+2+2+1+2+1

	Tetrads	Seven Units
p		4+1+2+1+2+1
52a	5 + 4 + 2	1+4+1+3+1+1
b		1+4+2+2+1+1
c		1+4+3+1+1+1
d		2+3+1+3+1+1
e		2+3+2+2+1+1
f		2+3+3+1+1+1
g		3+2+1+3+1+1
h		3+2+2+2+1+1
i		3+2+3+1+1+1
j		4+1+1+3+1+1
k		4+1+2+2+1+1
l		4+1+3+1+1+1
53a	6 + 2 + 2	1+5+1+1+1+1
b		2+4+1+1+1+1

	Tetrads	Seven Units
c		3+3+1+1+1+1
d		4+2+1+1+1+1
e		5+1+1+1+1+1
54a	6 + 2 + 3	1+5+1+1+1+2
b		1+5+1+1+2+1
c		2+4+1+1+1+2
d		2+4+1+1+2+1
e		3+3+1+1+1+2
f		3+3+1+1+2+1
g		4+2+1+1+1+2
h		4+2+1+1+2+1
i		5+1+1+1+1+2
j		5+1+1+1+2+1
55a	6 + 3 + 2	1+5+1+2+1+1
b		1+5+2+1+1+1

Tetrads		Seven Units
c		2+4+1+2+1+1
d		2+4+2+1+1+1
e		3+3+1+2+1+1
f		3+3+2+1+1+1
g		4+2+1+2+1+1
h		4+2+2+1+1+1
i		5+1+1+2+1+1
j		5+1+2+1+1+1
56a	7 + 2 + 2	1+6+1+1+1+1
b		2+5+1+1+1+1
c		3+4+1+1+1+1
d		4+3+1+1+1+1
e		5+2+1+1+1+1
f		6+1+1+1+1+1

IV.

	Pentads	Nine Units
1	2 + 2 + 2 + 2	1+1+1+1+1+1+1+1
2a	2 + 2 + 2 + 3	1+1+1+1+1+1+1+2
b		1+1+1+1+1+1+2+1
3a	2 + 2 + 2 + 4	1+1+1+1+1+1+1+3
b		1+1+1+1+1+1+2+2
c		1+1+1+1+1+1+3+1
4a	2 + 2 + 2 + 5	1+1+1+1+1+1+1+4
b		1+1+1+1+1+1+2+3
c		1+1+1+1+1+1+3+2
d		1+1+1+1+1+1+4+1
5a	2 + 2 + 3 + 2	1+1+1+1+1+2+1+1
b		1+1+1+1+2+1+1+1
6a	2 + 2 + 3 + 3	1+1+1+1+1+2+1+2
b		1+1+1+1+1+2+2+1
c		1+1+1+1+2+1+1+2

Pentads		Nine Units
d		1+1+1+1+1+2+1+2+1
7a	2 + 2 + 3 + 4	1+1+1+1+1+1+2+1+3
b		1+1+1+1+1+1+2+2+2
c		1+1+1+1+1+1+2+3+1
d		1+1+1+1+1+2+1+1+3
e		1+1+1+1+1+2+1+2+2
f		1+1+1+1+1+2+1+3+1
8a	2 + 2 + 4 + 2	1+1+1+1+1+1+3+1+1
b		1+1+1+1+1+2+2+1+1
c		1+1+1+1+1+3+1+1+1
9a	2 + 2 + 4 + 3	1+1+1+1+1+1+3+1+2
b		1+1+1+1+1+1+3+2+1
c		1+1+1+1+1+2+2+1+2
d		1+1+1+1+1+2+2+2+1
e		1+1+1+1+1+3+1+1+2

	Pentads	Nine Units
f		1+1+1+1+3+1+2+1
10a	2 + 2 + 5 + 2	1+1+1+1+1+4+1+1
b		1+1+1+1+2+3+1+1
c		1+1+1+1+3+2+1+1
d		1+1+1+1+4+1+1+1
11a	2 + 3 + 2 + 2	1+1+1+2+1+1+1+1
b		1+1+2+1+1+1+1+1
12a	2 + 3 + 2 + 3	1+1+1+2+1+1+1+2
b		1+1+1+2+1+1+2+1
c		1+1+2+1+1+1+1+2
d		1+1+2+1+1+1+2+1
13a	2 + 3 + 2 + 4	1+1+1+2+1+1+1+3
b		1+1+1+2+1+1+2+2
c		1+1+1+2+1+1+3+1
d		1+1+2+1+1+1+1+3

Pentads	Nine Units
e	1+1+2+1+1+1+2+2
f	1+1+2+1+1+1+3+1
14a 2 + 3 + 3 + 2	1+1+1+2+1+2+1+1
b	1+1+1+2+2+1+1+1
c	1+1+2+1+1+2+1+1
d	1+1+2+1+2+1+1+1
15a 2 + 3 + 3 + 3	1+1+1+2+1+2+1+2
b	1+1+1+2+1+2+2+1
c	1+1+1+2+2+1+1+2
d	1+1+1+2+2+1+2+1
e	1+1+2+1+1+2+1+2
f	1+1+2+1+1+2+2+1
g	1+1+2+1+2+1+1+2
h	1+1+2+1+2+1+2+1
16a 2 + 3 + 4 + 2	1+1+1+2+1+3+1+1

Pentads	Nine Units
b	1+1+1+2+2+2+1+1
c	1+1+1+2+3+1+1+1
d	1+1+2+1+1+3+1+1
e	1+1+2+1+2+2+1+1
f	1+1+2+1+3+1+1+1
17a 2 + 4 + 2 + 2	1+1+1+3+1+1+1+1
b	1+1+2+2+1+1+1+1
c	1+1+3+1+1+1+1+1
18a 2 + 4 + 2 + 3	1+1+1+3+1+1+1+2
b	1+1+1+3+1+1+2+1
c	1+1+2+2+1+1+1+2
d	1+1+2+2+1+1+2+1
e	1+1+3+1+1+1+1+2
f	1+1+3+1+1+1+2+1
19a 2 + 4 + 3 + 2	1+1+1+3+1+2+1+1

	Pentads	Nine Units
b		1+1+1+3+2+1+1+1
c		1+1+2+2+1+2+1+1
d		1+1+2+2+2+1+1+1
e		1+1+3+1+1+2+1+1
f		1+1+3+1+2+1+1+1
20a	2 + 5 + 2 + 2	1+1+1+4+1+1+1+1
b		1+1+2+3+1+1+1+1
c		1+1+3+2+1+1+1+1
d		1+1+4+1+1+1+1+1
21a	3 + 2 + 2 + 2	1+2+1+1+1+1+1+1
b		2+1+1+1+1+1+1+1
22a	3 + 2 + 2 + 3	1+2+1+1+1+1+1+2
b		1+2+1+1+1+1+2+1
c		2+1+1+1+1+1+1+2
d		2+1+1+1+1+1+2+1

	Pentads	Nine Units
23a	3 + 2 + 2 + 4	1+2+1+1+1+1+1+3
b		1+2+1+1+1+1+2+2
c		1+2+1+1+1+1+3+1
d		2+1+1+1+1+1+1+3
e		2+1+1+1+1+1+2+2
f		2+1+1+1+1+1+3+1
24a	3 + 2 + 3 + 2	1+2+1+1+1+2+1+1
b		1+2+1+1+2+1+1+1
c		2+1+1+1+1+2+1+1
d		2+1+1+1+2+1+1+1
25a	3 + 2 + 3 + 3	1+2+1+1+1+2+1+2
b		1+2+1+1+1+2+2+1
c		1+2+1+1+2+1+1+2
d		1+2+1+1+2+1+2+1
e		2+1+1+1+1+2+1+2

	Pentads	Nine Units
f		2+1+1+1+1+2+2+1
g		2+1+1+1+2+1+1+2
h		2+1+1+1+2+1+2+1
26a	3 + 2 + 4 + 2	1+2+1+1+1+3+1+1
b		1+2+1+1+2+2+1+1
c		1+2+1+1+3+1+1+1
d		2+1+1+1+1+3+1+1
e		2+1+1+1+2+2+1+1
f		2+1+1+1+3+1+1+1
27a	3 + 3 + 2 + 2	1+2+1+2+1+1+1+1
b		1+2+2+1+1+1+1+1
c		2+1+1+2+1+1+1+1
d		2+1+2+1+1+1+1+1
28a	3 + 3 + 2 + 3	1+2+1+2+1+1+1+2
b		1+2+1+2+1+1+2+1

Pentads	Nine Units
c	1+2+2+1+1+1+1+2
d	1+2+2+1+1+1+2+1
e	2+1+1+2+1+1+1+2
f	2+1+1+2+1+1+2+1
g	2+1+2+1+1+1+1+2
h	2+1+2+1+1+1+2+1
29a 3 + 3 + 3 + 2	1+2+1+2+1+2+1+1
b	1+2+1+2+2+1+1+1
c	1+2+2+1+1+2+1+1
d	1+2+2+1+2+1+1+1
e	2+1+1+2+1+2+1+1
f	2+1+1+2+2+1+1+1
g	2+1+2+1+1+2+1+1
h	2+1+2+1+2+1+1+1
30a 3 + 4 + 2 + 2	1+2+1+3+1+1+1+1

	Pentads	Nine Units
b		1+2+2+2+1+1+1+1
c		1+2+3+1+1+1+1+1
d		2+1+1+3+1+1+1+1
e		2+1+2+2+1+1+1+1
f		2+1+3+1+1+1+1+1
31a	4 + 2 + 2 + 2	1+3+1+1+1+1+1+1
b		2+2+1+1+1+1+1+1
c		3+1+1+1+1+1+1+1
32a	4 + 2 + 2 + 3	1+3+1+1+1+1+1+2
b		1+3+1+1+1+1+2+1
c		2+2+1+1+1+1+1+2
d		2+2+1+1+1+1+2+1
e		3+1+1+1+1+1+1+2
f		3+1+1+1+1+1+2+1
33a	4 + 2 + 3 + 2	1+3+1+1+1+2+1+1

	Pentads	Nine Units
b		1+3+1+1+2+1+1+1
c		2+2+1+1+1+2+1+1
d		2+2+1+1+2+1+1+1
e		3+1+1+1+1+2+1+1
f		3+1+1+1+2+1+1+1
34a	4 + 3 + 2 + 2	1+3+1+2+1+1+1+1
b		1+3+2+1+1+1+1+1
c		2+2+1+2+1+1+1+1
d		2+2+2+1+1+1+1+1
e		3+1+1+2+1+1+1+1
f		3+1+2+1+1+1+1+1
35a	5 + 2 + 2 + 2	1+4+1+1+1+1+1+1
b		2+3+1+1+1+1+1+1
c		3+2+1+1+1+1+1+1
d		4+1+1+1+1+1+1+1

TABLE II.

FAMILIES OF CHORD STRUCTURES

Chord Structures Classified Into Families

Arabic numerals indicate the numerical order of groups in Table I.

Original Structures	Derivative Structures	Triads
2	9	
3	16	
4	22	
5	27	
6	31	
7	34	
8	36	
11	17	

Original Structures	Derivative Structures	Triads
12	23	
13	28	
14	32	
15	35	
19	24	
20	29	
21	33	
26	30	

Original Structures	Derivative Structures	Tetrads
2	7 22	
3	12 37	
4	16 47	
5	19 53	
6	21 56	
9	13 38 24 31 41	
10	17 48 25 34 50	
11	20 54 26 36 55	
14	39 44	
15	18 49 40 46 52	
27	23 8	
29	32 42	
30	35 51	
45	43 33	

Original Structures	Derivative Structures	Pentads
2	5 11 21	
3	8 17 31	
4	10 20 35	
6	14 27 22 12 24	
7	16 30 32 9 19 34 23 13 26 18 33	

INDEX OF CHORDS AND SCALES

Chords		Scales
	I.	
Diads		Three Units
10		55
	II.	
Triads		Five Units
36		330
	III.	
Tetrads		Seven Units
56		462
	IV.	
Pentads		Nine Units
35		165
	Total:	
137		1012